The Electronic Kilogram[1]

Michael H. Kelley

National Institute of Standards and Technology, Gaithersburg, MD 20899-8112, USA

Abstract – **The kilogram is the only remaining base unit in the International System of Units (SI) whose definition is based on a single physical artifact rather than on fundamental properties of nature. Effects such as environmental contamination or material loss from surface cleaning are causing the "true" mass of the International Prototype Kilogram to drift (by about 5 parts in 10^8 per century), relative to sister prototypes. The equivalence of electrical and mechanical power provides a possible alternate measurement of mass in terms of other units that are based on fundamental quantum mechanical principles, such as the speed of light, the Josephson voltage, and the quantum Hall resistance. This provides a possible time-invariant definition of mass.**

I. INTRODUCTION

Though the term "Electronic Kilogram" probably sounds strange to anyone hearing it for the first time, it captures very well the long-term goal of one of the experimental efforts of the Electricity Division at NIST. That is, we are developing a measurement system with the ultimate goal of replacing the current definition of the unit of mass with one in which mass is derived from fundamental electrical measurements. Though this goal is relatively new, the experimental effort is not. The present work extends efforts that have for many years been used to fix the units of electrical measurements. It will also be seen, perhaps surprisingly, that the results of this experiment provide the experimental determination of several fundamental physical constants, such as the Planck constant.

To properly appreciate the significance of this work, it is useful first to step back and take a look at the present international system of units, *Le Système International d'Unités,* or the SI. The SI has been carefully designed to provide an internationally consistent system of units for all physical measurements. The SI [1] comprises seven "base" units: the meter (m), the kilogram (kg), the second (s), the ampere (A), the Kelvin (K), the mole (mol), and the candela (cd). All other measurement units can be derived from various combinations of these seven and are hence called "derived" units. For purposes of the present work we need only be concerned with the first four of these base units.

Of these four, one is distinctly different from the others: the kilogram. An essential difference is that the formal definition of the kilogram is the mass of one particular physical artifact, the International Prototype Kilogram (IPK), which was fabricated over 100 years ago and which permanently resides in a vault at the BIPM (International Bureau of Weights and Measures) outside of Paris, France. This is unsatisfactory for two principal reasons. First, the mass of any artifact, through a variety of mechanisms, drifts with time – including the mass of the IPK. Well, not quite. Since the IPK *defines* the unit of mass, it is the unit of mass that drifts so that the mass of the IPK is constant. The mass of everything else but the IPK drifts. Second, one basic principle of international standards is that they should be equally accessible to everyone. There is only one IPK, and it stays locked in its vault. About once every 50 years it is removed from its vault for comparison with other mass standards. Of course, to compare with the IPK, we have to send our mass standard to Paris for the comparison, during which time we do not have it for our own use.

By contrast, the meter, second, and ampere are defined by fundamental, presumably permanent and unalterable, physical properties of nature. Hence, they are in principle equally accessible to all.

The formal definition of the second, which derives from the observation that atomic transition frequencies are extremely reproducible and hence make extraordinarily good clocks, fixes exactly the frequency of one particular atomic transition. Specifically:

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.

---

[1] Contribution of the U.S. Government, not subject to U.S. copyright.

Similarly, the formal definition of the meter derives from the observation that the speed of light is constant, and fixes its value exactly. Specifically:

The meter is the length of the path traveled by light in vacuum during a time interval of 1 / 299 792 458 of a second.

The formal definition of the ampere uses the law of Biot-Savart to relate the current flowing in two wires to the electromagnetic force between them. This definition is based on the observation that the impedance of free space is a universal constant. Specifically:

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.

This definition fixes the value of the magnetic constant to be $\mu_0 = 4\pi \ 10^{-7}$ N A^{-2}. By extension, the electric constant is also fixed through Maxwell's equations to be $\varepsilon_0 = 1/(\mu_0 c^2) = 8.854 \ 1878 \ 17...\times 10^{-12}$ F m^{-1}.

II. "REALIZATION" VS. "REPRESENTATION" OF THE ELECTRICAL UNITS

In metrology-speak, the term "realization of a unit" generally refers to a sophisticated experiment, based on well-established physical principles, that produces the measurement unit in terms of its SI definition. Because such experiments are generally very difficult and often require a very long time (sometimes decades) to complete, they are typically carried out only in National Metrology Institutes, such as NIST or its sister organizations around the world (indeed, work similar to what is discussed here is carried out in several other Institutes, but the focus here will be on work at NIST). In general, an experimental program that realizes the unit for some measured physical quantity provides the highest level of accuracy and precision that is presently achievable for the measurement of that quantity. Though it represents the "best" measurement, it is seldom a convenient measurement for routine use. Hence, the realization of most units is accompanied by a practical representation of the unit. This representation is an experiment or artifact that is a reliable surrogate for the unit and that can be conveniently and routinely used for comparison with other measurement standards for the unit. The representation serves as a flywheel between the complete realizations of a unit.

In the case of the units of mass, length, and time, the SI definition is such that their experimental realizations are conceptually straightforward. Though the actual experiments require extreme care and continually push the limits achievable by present state-of-the-art metrology, they develop in expected ways from the formal definition. By contrast, the formal definitions of the electrical units do not lend themselves to such straightforward implementation. In the specific case of the ampere, it is very difficult both to fabricate two infinitely long and parallel wires and to measure the attractive force per unit of their length. Fortunately, one can develop an analogous definition, based on the impedance of free space through the electric rather than the magnetic constant, which leads to a convenient realization of the unit of capacitance, the farad. In 1956, Thompson and Lampard proved an elegant theorem in electrostatics which shows that, under very general conditions, a capacitor can be constructed whose capacitance does not depend on its physical dimensions, but whose capacitance per unit length can be calculated exactly [2]. In this calculable capacitor, the farad is realized through a single measurement of length. By a set of experimental measurements that is outside the scope of the present discussion, the unit of electrical resistance, the ohm, is realized by transferring the capacitive impedance of the calculable capacitor to the resistive impendence of a resistor, the frequency dependence of whose resistance is accurately known. It is interesting that all impedance measurements are traceable back to the SI not through the base unit of current, but through the derived unit of capacitance.

Because of the complexity of the chain of measurements required to connect the ohm with the farad, it is very fortunate that we have access to a very reliable practical representation of the unit of resistance based on the quantum Hall effect. In the integral quantum Hall effect, the Hall resistance of a two-dimensional electron gas exhibits constant plateaus as a function of applied magnetic field. The Hall resistance of the ith plateau is given by $R_H(i) = 1/i \ h/e^2$, where h is the Planck constant, e is the elementary charge of the electron. This particular combination of h and e are referred to as the von Klitzing constant, $R_K = h/e^2$. It is also interesting that the von Klitzing constant is closely related to one of the basic constants of physics, the fine structure constant, by $\alpha = \mu_0 c \ e^2/ 2h = \mu_0 c / 2 R_K$. As a result, we reach the remarkable conclusion that "calibrating" the quantized Hall resistance (QHR), that is linking its value to the SI through the farad, provides one of the most accurate experimental determinations of this basic fundamental constant [3].

It is important to note that the fundamental constants determining the quantum Hall resistance were not known sufficiently well to provide an accurate value of that

resistance. Rather, we use the measured value of that resistance to improve our knowledge of the fundamental constants. Further, it has been demonstrated that two independent QHR standards have the same resistance to a very high level of accuracy – better than one part in 10^{10}. That significantly exceeds the accuracy with which we can determine the von Klitzing constant, presently about 2.4 parts in 10^8. That is, we know they have the same resistance, but we are not completely sure what that resistance is.

A similar situation exists for the unit of voltage. However, before describing the realization the volt, we will describe its practical representation. In 1962, Brian Josephson gave a theoretical prediction for a remarkable behavior in a junction consisting of two superconductors separated by a thin insulating barrier [4]. In what has become known as the ac Josephson effect, such a junction can act as an ideal frequency-to-voltage converter. When driven at a microwave frequency f, the current-voltage dependence develops regions of constant voltage at values of $n f / K_j$, where n is a known integer and the Josephson constant K_j is given by $K_j = 2 e/h$, with h and e again the Planck constant and the electron charge. Today, voltage metrology around the world is based on large-scale series arrays of Josephson junctions, so-called Josephson Array Voltage Standards (JAVS), which produce a voltage determined only by the frequency of a driving microwave source and the Josephson constant.

The JAVS is an extraordinarily precise voltage source. The voltage outputs of two independent JAVS systems can be compared and demonstrated to agree within a few parts in 10^{10} [5]. However, as was the case for the quantum Hall resistance, the uncertainty in the present value of the Josephson constant, about 4 parts in 10^8, is too large for us to know with corresponding accuracy what the output voltage, in SI units, of the Josephson systems actually is.

The international metrology community agreed in 1990 on a way to deal with the unfortunate fact that the practical representations of the ohm and the volt could apparently support more reproducible measurements than could be specified by the underlying fundamental constants. Consensus values for both the von Klitzing and the Josephson constant were chosen. These are referred to, respectively, as R_{K-90} and K_{J-90}. Resistance and voltage measurements worldwide are referenced to QHR and JAVS systems that use these consensus values. It is important to note that while these measurements are very nearly equal to measurements reported in the SI units of resistance and voltage, they are not actually in SI units. We often refer to them as the "*1990*" units. For most practical applications, the difference is negligible. In high

precision metrology, however, the difference is extremely important and represents the heart of the present work.

III. The Watt Balance

The SI unit of voltage is realized in an experiment designed to compare the SI unit of power, the watt, as determined by both electrical and mechanical measurements. Specifically, the unit of voltage is defined to enforce the equivalence of electrical and mechanical power. That is, the Josephson constant is chosen to provide a unit of voltage that is consistent with the mechanical units.

To understand how one might connect electrical with mechanical measurement units, consider first a simple experiment for demonstrating a relationship between electromotive and mechanical force. An otherwise conventional mass balance is modified so that the gravitational force acting on a mass standard on one side of the balance is countered by a simple electromagnetic motor. That motor is a current loop immersed in an inhomogeneous magnetic field. One could in principle define the unit of electric current as that current required to exactly balance a specified mass standard on the balance. Unfortunately, this idea doesn't work very well for fairly obvious reasons. Neither the strength of the inhomogeneous field, nor its detailed shape, nor the geometry of the current loop, nor the position of the loop within the field can be determined with enough accuracy and precision to make this an acceptable method.

As a second possibility, consider a simple electric generator. A pickup coil is driven through a magnetic field. The open-loop voltage developed on the coil is measured as a function of the velocity with which the coil moves. One defines the unit of voltage as that voltage developed at a particular drive velocity. This idea fails for the same reason. Neither the strength of the field, nor its detailed shape, nor the geometry of the pickup coil, nor the position of the coil within the field can be determined with enough accuracy and precision.

We use a third possibility, one that was first proposed in 1976 by Kibble [6], that combines both measurements in such a way that the unknown quantities are exactly the same in both cases and hence need not be measured. The experiment proceeds in two steps, both using the same coil in a magnetic field. In the first step, the weighing step, the current, I, through the coil is used to balance the gravitational force, F_z, of a mass standard with mass m. The local acceleration of gravity, g, is also carefully measured. The balance condition is given as $F_z = m g = G I$. G is an unknown geometry factor that depends on the vertical magnetic flux gradient of the

magnetic field and the coil geometry. In the second step, the velocity step, the coil is driven through the field at a fixed velocity, v_z, while the open-loop voltage, U, of the coil is measured. The measured voltage is given by $U = G v_z$. Here G is again the unknown geometry factor. It is important to realize, however that this is exactly the *same* factor that appears in the balance condition for the weighing step. That is, $G = F_z/I = U/v_z$. This expression, rewritten as $F_z v_z = U I$, can be recognized as a simple statement of the equality of mechanical power and electrical power.

Rather than measure current directly, we measure the voltage drop generated by that current across a standard resistor. Both this voltage and that developed across the pickup coil in the velocity step are measured with respect to the Josephson voltage, and the resistor is calibrated against the quantum Hall resistance

From our measurements, we determined the mechanical power in the SI unit of power, $P_{Mech} \{W_{SI}\}$. Similarly, we determined the electrical power in the *1990* units of power, $P_{Elec} \{W_{90}\}$. Since the electrical and mechanical power are the same, any difference between the electrical power in *1990* units and in SI units must be due to an inaccurate assignment of K_{J-90} and/or R_{K-90}. We can use the ratio of the power measured in these two units, R_{90}, to correct our assignment of these constants. That is $P_{Elec} \{W_{SI}\} = R_{90} P_{Elec} \{W_{90}\}$. From our equations describing the QHR and JAVS systems above, we know that $P_{Elec} \{W_{SI}\} = h/4 \ n^2 f^2 i$ where n and i are integers known from the operation of Josephson and QHR systems and f is a precisely measured frequency. Similarly, the electrical power is $P_{Elec} \{W_{90}\} = n^2 f^2 i \ (K^2_{J-90} R_{K-90})^{-1}$. The n, i, and f have the same values as before and can be eliminated from the expression. Combining these expressions, we find that $h/4 = R_{90} (K^2_{J-90} R_{K-90})^{-1}$

Thus, any mismatch between the *1990* and the SI unit of power can be used to determine an improved value for the Planck constant that is used to determine K_J and/or R_K. Our present measured value of this constant is $h = 6.626\ 068\ 76(52) \times 10^{-34}$ J s, which is accurate to about 8.7 parts in 10^8 [7].

IV. THE ELECTRONIC KILOGRAM

Thus far we have described how the equivalence of electrical and mechanical power defines electrical SI units that are consistent with the mechanical units, specifically with the mass of the IPK. The ultimate goal of our program is to turn that around – to define the kilogram such that the mass of the IPK is consistent with the

electrical units. Up until now, the weak link in the chain connecting the electrical and mechanical units has been the experimental apparatus used to realize the unit of power. We are now concentrated on improving the experimental system to reduce that uncertainty by about an order of magnitude, to one part in 10^8.

When that level of accuracy is achieved, we expect to be able monitor the long-term drift in the unit of mass. That is, because all other measurements would be based on unchanging physical properties of nature, we would attribute any future apparent drift in the measured value of the Planck constant to an actual drift in the mass unit.

At that time, a possible redefinition of the SI could be considered. That is, one could fix the value of the Planck constant, and define the unit of mass with reference to the electrical units – an electronic kilogram [8].

REFERENCES

[1] B. N. Taylor, editor, *The International System of Units (SI)*, NIST Special Publication 330, Washington, DC: US GPO, 1991.

[2] A.M. Thompson and D.G. Lampard, "A new theorem in electrostatics and its application to calculable standards of capacitance," *Nature*, Vol 177, p. 888, 1956.

[3] A. Jeffery, R.E. Elmquist, J.Q. Shields, L.H. Lee, M.E. Cage, S.H. Shields, and R.F. Dziuba, "Determination of the von Klitzing constant and the fine-structure constant through a comparison of the quantized Hall resistance and the ohm derived from the NIST calculable capacitor", *Metrologia*, Vol 35, pp. 83-96, 1998.

[4] B.D. Josephson, "Possible new effects in superconductive tunneling," *Physics Letters*, Vol 1, pp. 251-253, 1962.

[5] C. A. Hamilton, "Josephson voltage standards", Review of Scientific Instruments, Vol 71, pp. 3611-3623, Oct. 2000.

[6] B. P. Kibble, *Atomic Masses and Fundamental Constants*, Vol. 5, p. 545, edited by J.H. Sanders and A.H. Wapstra, New York: Plenum, 1976

[7] E.R. Williams, R.L. Steiner, D.B. Newell, P.T. Olsen, "Accurate measurement of the Planck constant", *Physical Review Letters* Vol 81, p. 2404, 1998.

[8] Barry N. Taylor, "The possible role of the fundamental constants in replacing the kilogram", *Metrologia*, Vol. 40, pp. 86-91, 1994.